

# Analysis of Capacitive Heat Exchangers

## Part II

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*The performance of a simple capacitive heat exchanger is analyzed. The shooting method for solving the governing equations is presented in detail. Simplified linear and nonlinear models are analyzed using this method and it is shown that capacitive heat exchangers are more efficient than conventional ones.*

### I. Introduction

Heat exchangers are important components of most industrial facilities. In the Deep Space Network they have many uses ranging from the heat exchangers that serve the hydrostatic oil bearings to those employed in the heating, ventilating and air conditioning system. Because of the large number of heat exchangers that exist throughout the DSN and because of the vital functions that they perform, it is desirable to seek improvements in their performance. To this end, the Advanced Engineering Group of the DSN Engineering Section is continuously investigating advanced concepts in heat exchangers.

The focus of one such investigation has been to analyze the theoretical performance of capacitive heat exchangers. These are two-phase, counter flow units, consisting of a stream of initially "hot" particles falling through an ascending stream of "cold" gas. The particles may be either solid or they can be liquid droplets. The potential advantages of such an arrangement are threefold. First, the two streams are in direct contact with each other. This eliminates the resistance to heat transfer which occurs when the hot and cold streams are separated by a tube wall. Second, the contact area between the two streams is greatly magnified because the heat exchange takes place on the entire surface of each particle. Finally, friction between the two streams is less than when a solid interface is present.

The equations that govern the heat transfer in such a heat exchanger have been laid out in Part I of this article (Ref. 1). It was pointed out that the equations were based on some simplifying assumptions, assumptions which were necessary because of the complexity of the physical phenomena that occur in capacitive heat exchangers. In particular it was assumed that:

- (1) The flow can be considered as one-dimensional and steady.
- (2) The discrete phase is composed of identical spherical particles of volume  $V_D$ .<sup>1</sup> The volume fraction occupied by the particles,  $\kappa$ , is assumed to be small and therefore the interactions between particles can be neglected.

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<sup>1</sup> A list of symbols appears at the end of the article.

- (3) The force interaction between the two streams is due to the drag between them and to gravitational and buoyancy effects. Electric and spin forces are neglected.
- (4) Radiative heat transfer can be neglected, convection being the main mode of heat transfer between the spheres and the gas.

Following these assumptions, the governing laws for a capacitive heat exchanger were shown to be represented by a set of 13 equations and 13 unknowns. This set is reproduced in Table 1.

In this second part of the report, the main purpose is to present methods for solving the equations and to discuss the results obtained. The methods of solution are numerical in nature and their application necessitates a careful look at the equations. For example, these numerical methods require parameterization and taking the derivative of complicated functional relationships. Part II also presents a sensitivity analysis of the equations. In addition some clarifications are made to Part I of the paper. Finally, the performance of capacitive heat exchangers is compared to that of conventional counterflow exchangers and areas of future investigation are sketched out.

## II. The Boundary Value Problem and Methods of Solution

### A. The Normalized Equations

The equation set given in Table 1 consists mainly of conservation equations for mass, momentum and energy. In each of these pairs, the first equation represents conservation of the continuous phase and the second represents conservation of the dispersed phase. In addition, functional relationships are needed for the viscosity coefficient, the heat capacities of the two phases, and for the dispersed phase density. All these relationships are functions of the respective phase temperature. The state equation and two empirical correlations which are needed for the drag coefficient and for the Nusselt numbers complete the set. The last two equations are based on experiments reported in the literature (Refs. 2, 3). The initial equations chosen are:

$$N_{Nu} = (1.14 \cdot 10^{-3}) * (N_D V_D)^{-0.5984} (N_{Re})^{0.8159} \quad (1)$$

$$\overline{C_d} = 27.0 N_{Re}^{-0.84} \quad (2)$$

The coefficients in these relations, especially for the drag equation have been known to vary greatly from one investigation to another.

It was pointed out in Part I that in order to solve the system of equations there must not be any derivative implicitness. This means that one must be able to write the equations such that only one derivative occurs in each equation. In Part I it was suggested that this can be accomplished by assuming that

$$\frac{dN_D}{dy} = \text{constant}$$

which is equivalent to

$$N_D = N_D \Big|_{y=0} \left( 1 + \epsilon \frac{y}{H} \right)$$

However, this assumption is not the only one that can eliminate derivative implicitness. Instead, it is simpler to assume that  $\rho_D$  is a constant. Physically this is plausible because the density of the particle material will not change substantially within the temperature ranges of the problem. This is especially so when the particle material is a silicate such as the glass used in the model. From a mathematical point of view, the assumption that  $\rho_D$  is a constant reduces the number of unknowns and therefore the number of equations in the set. At the same time, the derivative implicitness is also eliminated.

Before the equations are solved, it is first necessary to normalize them. This is important when comparisons are made with a standard heat exchanger and when sensitivity analyses are to be performed. In these cases, normalization can unify the results by relating them to common and dimensionless scales. For the problem at hand, one must normalize with respect to known quantities. These known quantities are the values that the variables take at the boundaries of the heat exchanger. For the counter flow configuration studied here, it seems reasonable to assume that the magnitude of some of the variables is known only at one end of the heat exchanger, while the magnitude of the other variables is known at the other end. For example, it is logical to assume that one knows the entering air temperature but not the exit temperature. Conversely, the temperature of the beads may be known at the top, but not at the bottom of the heat exchanger (Fig. 1).

The normalized variables are given in Table 2. Note that in the case of velocities and temperatures, only the dispersed phase is used for normalization, even though the values that the continuous phase variables take at the bottom of the heat exchanger are also known, e.g.,  $T_c$  at  $y = 0$ . However, in these cases, by using only the values of the dispersed phase, the normalized equations become simpler. Note also that, together with  $\rho_D$  a constant, it was also assumed that  $c_{p_C}$  and  $c_{p_D}$  are also constant; the physical justification for all these assumptions is the same. The gravitational acceleration is normalized by its value at sea level  $g_0$ , and for all practical purposes  $g^* = 1$ . The density and the pressure of the continuous phase can be normalized differently than shown in Table 2. For example, one can set  $p^* = p/(1/2 \rho_C v_C^2)$  and  $\rho_C^* = \rho_C/\rho_D$ . These formulations, when tried out, yield very complicated algebraic equations and create serious scaling problems in the boundary conditions. Scaling problems such as these can prevent the convergence of the numerical solution. Finally, by using correlations of the form

$$N_{Nu} = m (N_D V_D)^n N_{Re}^q \quad (3)$$

and

$$\overline{C_d} = a N_{Re}^b \quad (4)$$

one obtains the following set of dimensionless equations:

$$\frac{dv_C^*}{dy^*} (\rho_C^* (1 - \sigma_1 N_D^*)) + \frac{d\rho_C^*}{dy^*} (v_C^* (1 - \sigma_1 N_D^*)) - \frac{dN_D^*}{dy^*} (\sigma_1 \rho_C^* v_C^*) = 0 \quad (5)$$

$$v_D^* \frac{dN_D^*}{dy^*} + N_D^* \frac{dv_D^*}{dy^*} = 0 \quad (6)$$

$$v_C^* \frac{dv_C^*}{dy^*} + \sigma_7 \frac{dp^*}{dy^*} \left( \frac{1}{\rho_C^* (1 - \sigma_1 N_D^*)} \right) = -g^* \sigma_2 - \frac{3}{8} a \sigma_1 \sigma_3^b \left( \frac{\rho_C^* |v_C^* - v_D^*| r_D^*}{\mu_C^*} \right)^b \frac{N_D^*}{r_D^*} (v_C^* - v_D^*) |v_C^* - v_D^*| \quad (7)$$

$$v_D^* \frac{dv_D^*}{dy^*} = \frac{3}{8} a \sigma_3^b \left[ \frac{\rho_C^* |v_C^* - v_D^*| r_D^*}{\mu_C^*} \right]^b (1 - \sigma_1 N_D^*) \sigma_8 \frac{\rho_C^* (v_C^* - v_D^*) |v_C^* - v_D^*|}{r_D^*} - \sigma_2 (1 - \sigma_8 \rho_C^*) \quad (8)$$

$$\sigma_4 \rho_C^* v_C^* c_{p_C}^* \frac{dT_C^*}{dy^*} = \frac{3}{2} \frac{k_C^*}{r_D^{*2}} m \frac{(\sigma_1 N_D^*)^{n+1}}{1 - \sigma_1 N_D^*} \sigma_3^q \left( \frac{\rho_C^* |v_C^* - v_D^*| r_D^*}{\mu_C^*} \right)^q (T_D^* - T_C^*) \quad (9)$$

$$\sigma_4 v_D^* \frac{dT_D^*}{dy^*} = -\frac{3}{2} \frac{k_C^*}{r_D^{*2}} m (\sigma_1 N_D^*)^n \sigma_3^q \left( \frac{\rho_C^* |v_C^* - v_D^*| r_D^*}{\mu_C^*} \right)^q (T_C^* - T_D^*) \quad (10)$$

$$\sigma_9 \frac{dp^*}{dy^*} = \frac{d\rho_C^*}{dy^*} + \frac{dT_C^*}{dy^*} \quad (11)$$

$$\mu_C^* = \beta_1 T_C^* + \beta_2 \quad (12)$$

$$k_C^* = \beta_3 T_C^* + \beta_4 \quad (13)$$

The  $\sigma$ 's are dimensionless quantities that are made up of constants and boundary values. They are defined as follows

$$\sigma_1 = \frac{4}{3} \pi r_D^{*3} H^3 N_D \big|_{y=H} \quad (14)$$

$$\sigma_2 = \frac{g_0 H}{v_D^2 \big|_{y=H}} \quad (15)$$

$$\sigma_3 = \frac{2\rho_C \big|_{y=0} v_D \big|_{y=H} H}{\mu_C \big|_{y=0}} \quad (16)$$

$$\sigma_4 = \frac{\rho_C \big|_{y=0} v_D \big|_{y=H}^c P_D H}{k_C \big|_{T_D(H)}} \quad (17)$$

$$\sigma_7 = \frac{P_C \big|_{y=0}}{\rho_C \big|_{y=0} v_D^2 \big|_{y=H}} \quad (18)$$

$$\sigma_8 = \frac{\rho_C \big|_{y=0}}{\rho_D} \quad (19)$$

$$\sigma_9 = \frac{\rho_C \big|_{y=0}}{\rho_C \big|_{y=0} R T_D \big|_{y=H}} \quad (20)$$

Two of these quantities are easily recognizable,  $\sigma_3$  is in the form of a Reynolds number and  $\sigma_2$  in the form of a reciprocal Froude number. The first quantity,  $\sigma_1$ , represents the volume fraction  $\kappa$ , occupied initially, i.e., at  $y = H$  by the particles.  $\sigma_4$  and  $\sigma_5$  result from the way the variables are normalized. The last two equations, (19) and (20), are approximations to the actual functional relationships. The data for these relationships is illustrated in Figs. 2 and 3 (Ref. 4). The values for the constants are

$$\beta_1 = 2.004 \quad \beta_2 = 0.3710$$

$$\beta_3 = 0.8994 \quad \beta_4 = 0.1133$$

and, to simplify the notation

$$v_D^*|_{y=H} = v_D^*(H); T_D^*|_{y=H} = T_D^*(H); N_D^*|_{y=H} = N_D^*(H) \quad (21)$$

$$v_C^*|_{y=0} = v_C^*(0); T_C^*|_{y=0} = T_C^*(0); \rho_C^*|_{y=0} = \rho_C^*(0); \rho_C^*|_{y=0} = \rho_C^*(0) \quad (22)$$

## B. The Shooting Method

Equations (5) through (13), together with boundary conditions (21) and (22) form a two-point boundary value problem. When dealing with a system of  $n$  first order differential equations, the classical form of a two-point boundary value problem is

$$\dot{y}_i = f_i(y_1, y_2, \dots, y_n, t), \quad i = 1, 2, \dots, n \quad (23)$$

$$y_i(t_0) = C_i, \quad i = 1, 2, \dots, r \quad (24)$$

$$y_{i_m}(t_f) = C_{i_m}, \quad m = 1, 2, \dots, n-r \quad (25)$$

In this system,  $y_1, y_2, \dots, y_n$  are the dependent variables and  $t$  is the independent variable;  $f_i$  can be either a linear or a nonlinear function. The boundary conditions are separated here into initial conditions (24) and final conditions (25).

For a linear system of equations, analytical solutions can be found relatively easily. Such simple systems can be written as

$$\dot{y} = A y \quad (26)$$

where  $y$  is the vector  $y_1, y_2, \dots, y_n$ . The analytical solution consists of finding the eigenvalues and eigenvectors of  $A$  and then of constructing the matrix  $e^{tA}$ . The boundary conditions are then applied to yield the complete solution. When the equations are nonlinear, such as in the present case, the system can no longer be written as in Eq. (26) and, generally, numerical methods must be used to solve the problem. These methods are of two types: finite differences and shooting methods.

Finite difference methods basically replace the derivatives by finite differences; this has two important practical results. First, the ordinary differential equations are converted into a set of algebraic equations. Second, at the boundaries of the domain of interest, the boundary conditions are directly incorporated into the equations. Thus, finite difference methods have the advantage that they "hold" the entire solution at once. When the original differential equations are nonlinear, the resulting algebraic finite difference equations are also nonlinear. Theoretically, this does not present a problem as these algebraic equations can be solved by standard techniques such as the Newton-Raphson method. However, finite difference solutions of nonlinear boundary value problems do have certain disadvantages. The main problem with the Newton-Raphson method is to find a good starting guess. Therefore, when a reasonably good initial estimate for the  $y$  vector is not available or when the equations are strongly nonlinear, the finite differences method may not converge or may not give a unique solution (Refs. 5, 6).

An alternative to finite differences is the so-called shooting method (Refs. 6, 7, 8). The main idea behind the method is illustrated in Fig. 4. The missing conditions at one of the boundary points (Point "a" in the figure) are assumed and the equations are then integrated forward toward the other boundary point (Point "b" in the figure). When the solution reaches this "end" boundary point, it is compared to the actual conditions there, the initial guesses are adjusted and the problem is solved again. If the solution of the equation is well behaved, it is possible to use a linear interpolation for the missing initial conditions. Thus, a systematic computational scheme can be devised so that after an initial bracketing of the "target," the next "shots" land closer and closer to it until a specified tolerance is reached. Therefore the shooting method reduces the boundary value problem to a repeated iteration of an initial value problem. In general, such shooting solutions are faster than finite ones.

The method described above is called simple shooting and it too can have certain disadvantages. The main problem is that, quite often, "the differential equations are so unstable that they "blow up" before the initial value problem can be completely integrated. This can occur even in the face of extremely accurate guesses for the initial values" (Ref. 9). For such cases, an alternative procedure has been devised which is a sort of compromise between the simple shooting and the finite differences method. This is the multiple shootings method and will be the one used in the present case.

Multiple shooting is often successful in solving nonlinear problems because it limits the "blowing up" of the equations. The method starts by subdividing the integration interval by  $M$  nodes. For example, if the starting boundary point is at  $t_i$  and the end boundary point is at  $t_f$ , the new subdivisions will be at  $t_1, t_2, \dots, t_{M-1}, t_M, t_{M+1}, \dots, t_{M-1}, t_M$ , where  $t_1 = t_i$  and  $t_M = t_f$  (see Fig. 5). Consider now the initial value problem given by the original equation

$$\dot{y} = f(y, t), y = (y_1, y_2, \dots, y_N) \quad (27)$$

and by the set of initial conditions:

$$y(t_K) = S_K, t \in [t_K, t_{K+1}], K = 1, 2, \dots, M-1 \quad (28)$$

Let the solution of this initial value problem be denoted by  $y(t_K, S_K)$ . Note that  $S_K$  is unknown. The original boundary value problem can be recovered if a  $S_K$  vector is found such that it satisfies the following two sets of conditions:

- (1) Continuity conditions. For each adjacent pair of intervals, the solution must agree at their common node,  $t_M$ . These conditions can be written as:

$$F_K(S_K, S_{K+1}) = y(t_{K+1}; t_K, S_K) - S_{K+1} = 0 \quad (K = 1, 2, \dots, M-2) \quad (29)$$

- (2) Boundary conditions. At the ends of the interval the initial value problem must satisfy the boundary conditions of the original boundary value problem. These boundary conditions can be written as

$$r[y(a), y(b)] = 0 \quad (30)$$

and therefore the equations are:

$$F_{M-1}(S_1, S_{M-1}) = r[S_1, y(t_M; t_{M-1}, S_{M-1})] = 0 \quad (31)$$

The continuity and boundary conditions result in the following set of  $N(M-1)$  nonlinear equations

$$F(S) = \begin{pmatrix} F_1(S_1, S_2) \\ \vdots \\ F_{M-2}(S_{M-2}, S_{M-1}) \\ F_{M-1}(S_1, S_{M-1}) \end{pmatrix} = 0 \quad ; \quad S = \begin{pmatrix} S_1 \\ \vdots \\ S_{M-1} \end{pmatrix} \quad (32)$$

These equations can be solved iteratively by the Newton-Raphson method. What has been accomplished so far is that the original problem has been broken down into a number of smaller interval problems. The advantage is that these smaller intervals "can be made short enough so that the intergration will end before any instability takes over" (Refs. 9, 11).

Stoer and Bulirsch (Ref. 10) show by means of examples that "... even for simple linear separated boundary-value problems (difference) methods and variational methods are feasible ... only if the solution need not be computed very accurately. For the

treatment of nonlinear boundary-value problems for ordinary differential equations, the only feasible methods, effectively, are the multiple shooting method and its modifications.”

The multiple shooting method used here is incorporated into a computer code available from IMSL.<sup>2</sup> This routine is called DTPTB. The use of the Newton-Raphson method to solve (32) requires the computation of a Jacobian. In particular, the subroutine FCNJ (see Appendix A) computes  $\partial Y_j' / \partial Y_i$ . Because the expressions for  $Y_j'$  are algebraically quite complicated, analytical calculations of the derivatives are prone to error. The author was fortunate to have made available to him a new Fortran code, PDGEN, a partial derivative generator.<sup>3</sup> The input code of PDGEN consists of arithmetic assignment statements which define the functions to be differentiated. If the problem includes functions not in the PDGEN library, further defining statements must be used. For example, in the present case the function  $|v_C^* - v_D^*|$  appears very often. The ABS function is not built in in PDGEN; therefore it is necessary to use

$$\text{SGNFUN}(U) = \text{SIGN}(1., U)$$

$$\text{ABS}'(Z) = \text{SGNFUN}(Z)$$

(The first line is needed because the code does not accept a definition of a derivative as a function of two or more arguments.) PDGEN proceeds to break down the given function into temporary variables  $T1, T2, \dots$  and to give the results as

$$D(i, j) = f(T1, T2, \dots)$$

There are some limitations on how many  $T$  statements can be generated; this was not found to be a serious impediment in the present problem.

It turns out that even the multiple shootings method requires some help in ensuring its convergence. Again, the problem has to do with initial choices: on how to choose the intermediate points  $t_K$  for the set of initial value problems (27-28). The IMSL subroutine employs a parameterization process based on introducing into the problem a parameter  $\alpha$ . This parameter is introduced so that for  $\alpha = \alpha_0 = 0$ , the problem is relatively simple; usually this means a linear problem. The solution of the simpler problem furnishes an adequate guess for the next initial value problem. For this new problem,  $\alpha = \alpha_0 + \epsilon$  where  $\epsilon$  is a “small” number dependent on the number of iterations. The iterations procedure continues until  $\alpha = 1$ , at which point the original nonlinear problem has been recovered.

In the present case, the following scheme was considered.<sup>4</sup> Let the system of equations be written as

$$\frac{dY_i}{dy} = f_i(\mathbf{Y}) \quad , \quad \mathbf{Y} = Y_1, \dots, Y_N \quad (33)$$

Consider a first approximation for this equation. Let this first approximation be based on expanding the right-hand side of (33) into a Taylor series and retaining only the linear terms of the series. Then this first approximation can be written as:

$$\frac{dY_i^{(0)}}{dy} = \left. \frac{\partial f_i}{\partial Y_j} \right|_{y=0} (Y^{(0)}(y) - Y^{(0)}(0)) + f_i(Y(0)) \quad (34)$$

<sup>2</sup>IMSL Inc. (International Mathematical and Statistical Library, 7500 Bellaire Blvd., Houston, TX 77036). A library reference manual, available to IMSL subscribers, details the various subroutines used in the solution of two-point boundary value problems.

<sup>3</sup>The PDGEN code was designed and implemented by Dr. Webb Miller, UC Santa Barbara. The program has been implemented for JPL use by K. Stewart, E. Ng and C. Lawson for the JPL Software Development Section.

<sup>4</sup>The author is indebted to F. Krosh for suggesting this scheme.

where the superscript (0) denotes the "zeroth" value in a series of parameterizations. Using the parameter  $\alpha$ , this series can be written as

$$\frac{dY_i^{(\alpha)}}{dy} = (1 - \alpha) \left[ \frac{\partial f^{(\alpha)}}{\partial Y} \Big|_{y=0} (Y^{(\alpha)}(y) - Y^{(\alpha)}(0)) + f_i(Y^{(\alpha)}(0)) \right] + \alpha f_i(Y^{(\alpha)}) \quad (35)$$

where

$$\frac{\partial f^{(\alpha)}}{\partial y} = \left[ (1 - \alpha) \frac{\partial f}{\partial Y} \Big|_{y=0} + \alpha \frac{\partial f}{\partial Y} \Big|_{y=y} \right] \quad (36)$$

Note that when  $\alpha = 0$ , the first approximation (Eq. 34) is obtained, and when  $\alpha = 1$ , the original nonlinear problem (Eq. 33) is recovered. Therefore, for each  $0 < \alpha < 1$ , the code has a starting guess based on the previous  $\alpha$ .

### III. Analysis of Results for a Simplified Problem

#### A. Linear Case

Kays and London (Ref. 12) have introduced the concept of heat exchanger effectiveness. This is defined as the ratio between the actual heat transferred and the theoretical maximum that could be transferred. The effectiveness is a function of the hot and cold fluid capacity rates and of the "heat transfer size" of the exchanger. The heat transfer size is often labeled as the number of heat transfer units (NTU) and is given by the formula

$$NTU = \frac{1}{C_{min}} \int_0^A U dA$$

where  $U$  is the overall conductance and  $A$  is the same transfer area as used in the definition of  $U$ .  $C_{min}$  is the smaller of the hot and cold capacity rates, i.e., of  $(\dot{m}C_P)_{hot}$  and  $(\dot{m}C_P)_{cold}$  respectively. For a counterflow heat exchanger such as the one considered here:

$$\epsilon = \begin{cases} \frac{T_{hot,in} - T_{hot,out}}{T_{hot,in} - T_{cold,in}} & \text{if } C_{hot} = C_{min} \\ \frac{T_{cold,out} - T_{cold,in}}{T_{hot,in} - T_{cold,in}} & \text{if } C_{cold} = C_{min} \end{cases} = \frac{1 - e^{-NTU \left(1 - \frac{C_{min}}{C_{max}}\right)}}{1 - \left(\frac{C_{min}}{C_{max}}\right) e^{-NTU \left(1 - \frac{C_{min}}{C_{max}}\right)}} \quad (37)$$

Therefore, the efficiency  $\epsilon$  can be readily determined from a knowledge of the temperatures at the entrance and exit of the exchanger. Indeed, if the quantities describing the capacity rates and the heat transfer coefficient are approximated as constants (i.e., not functions of temperature), only two equations are needed to solve for the efficiency; the two equations are the temperature equations (T1.5) and (T1.6) Table 1.

Let

$$C_1 \equiv \frac{h}{\rho_c v_c c_{Pc}} \left( \frac{4\pi N_D r_D^2}{1 - N_D V_D} \right) = N_{STc} \cdot \frac{N_D V_D}{1 - N_D V_D} \cdot \frac{3}{r_D}, (C_1 > 0) \quad (38)$$



and

$$C_2 \equiv \frac{3h}{\rho_D v_D c_{pD} r_D} = N_{ST_D} \cdot \frac{3}{r_D}, (C_2 > 0) \quad (39)$$

where  $C_1$  and  $C_2$  are constants independent of temperatures. Therefore, in this case, the performance of the heat exchanger can be analyzed by solving the much simpler boundary value problem:

$$\begin{pmatrix} T_c \\ T_D \end{pmatrix} = \begin{pmatrix} -C_1 & C_1 \\ -C_2 & C_2 \end{pmatrix} \begin{pmatrix} T_c \\ T_D \end{pmatrix} \quad (40)$$

subject to the boundary conditions:

$$\left. \begin{aligned} y = 0, & \quad T_c = T_c(0) \\ y = H, & \quad T_D = T_D(H) \end{aligned} \right\} \quad (41)$$

Based on the eigenvalues

$$\lambda_1 = 0; \lambda_2 = C_2 - C_1$$

the solutions are:

$$T_c = T_c(0) - \frac{T_c(0) - T_D(H)}{1 - \frac{C_2}{C_1} e^{(C_2 - C_1)H}} \left( 1 - e^{(C_2 - C_1)y} \right) \quad (42)$$

$$T_D = T_c(0) - \frac{T_c(0) - T_D(H)}{1 - \frac{C_2}{C_1} e^{(C_2 - C_1)H}} \left( 1 - \frac{C_2}{C_1} e^{(C_2 - C_1)y} \right) \quad (43)$$

The physical meaning of the quantities  $C_1$  and  $C_2$  can be readily obtained by constructing the ratio  $C_2/C_1$ . This ratio is:

$$\frac{C_2}{C_1} = \frac{\rho_c v_c c_{pC}}{\rho_D v_D c_{pD}} * \frac{1 - N_D V_D}{N_D V_D} \quad (44)$$

and it represents the ratio between the heat capacities of the two streams. The magnitude of this ratio,  $C_{cold}/C_{hot}$ , can be less than, equal to or greater than 1. Changes in this ratio about a value of 1 affect the curvature of  $T_c$  and  $T_D$  variations with respect to distance. This is illustrated in Fig. 6. When  $C_2/C_1 = 1$ , the capacities are said to be "matched." In this case the temperatures form straight parallel lines with respect to distance; i.e., the difference  $T_D - T_c$  is a constant.

It was mentioned previously that  $C_1$  and  $C_2$  are assumed to be independent of temperature. For the cases analyzed here, the physical properties of glass and air were evaluated at 1000 K. The performance of the heat exchanger does not change significantly if the properties are evaluated at a different temperature. This is illustrated in Fig. 7, where the axial variations of temperatures are plotted first for physical properties evaluated at 1000 K and then at 1600 K. Therefore, for the rest of the linear analysis the physical properties are kept constant and evaluated at 1000 K (Ref. 13). Furthermore, in most standard heat exchangers as well as in capacitive ones (Refs. 14, 15), the velocity ratio and the heat capacity ratio are both of order 1. For the present linear analysis the following values were chosen as fixed:

$$\begin{aligned} v_C &= 2.25 \text{ m/sec} & v_D &= 2.0 \text{ m/sec} \\ c_{P_C} &= 0.27 \text{ cal/g, K} & c_{P_D} &= 0.30 \text{ cal/g, K} \end{aligned}$$

Under these conditions, the only change in the  $C_2/C_1$  ratio can occur due to a change in  $N_D V_D$  only. This is illustrated in Fig. 8, where  $\kappa = N_D V_D$  is plotted vs  $C_2/C_1$ . From this figure, note the unique value  $\bar{\kappa} = 1.6 \cdot 10^{-4}$ , which for the physical properties chosen gives the matched capacity rates condition.

To determine the efficiency  $\epsilon$  of a heat exchanger, Eq. (37), the heat capacity ratio  $C_R = C_{min}/C_{max}$  and the NTU must be computed. For various values of  $\kappa$ , this is illustrated in Fig. 9. As predicted from the previous figure,  $C_R$  reaches a value of 1.0 at  $\bar{\kappa}$  and then falls off for both increasing and decreasing  $\kappa$ . The NTU can be computed as

$$NTU = \frac{(h) 4\pi r_D^2 N_D}{C_{min}} \quad (45)$$

The dependency of NTU on  $\kappa$  is illustrated in Fig. 9 for two typical particle radii,  $r_D = 0.1$  cm and  $r_D = 0.2$  cm. The shape of this curve is due mainly to the changes in the convective film coefficient  $h$  with respect to  $\kappa$ ; for various ranges of  $\kappa$  one can choose from different correlations of the Nusselt number. (These correlations are given in Ref. 2). Given either  $C_R$  and NTU or  $C_{min}$ ,  $C_{max}$  and the boundary temperatures, one can determine the efficiency of the heat exchanger as given in Eq. (37). The efficiency as a function of  $\kappa$  is plotted in Fig. 10. As expected, at  $\bar{\kappa}$  the value of the efficiency is the lowest,  $\epsilon = 0.79$ . On either side of  $\bar{\kappa}$ , the efficiency rises rapidly, reaching 99% for  $\kappa \geq 4 \cdot 10^{-4}$  and  $\kappa \leq 2 \cdot 10^{-5}$ . It is instructive to compare the capacitive heat exchanger efficiency with that of a conventional counterflow heat exchanger. One way to do this is to look at Fig. 11, which is adapted from Ref. 12. From the previous two figures, for a capacitive heat exchanger at matched capacity rates, the efficiency occurs at NTU = 1.5 and is 79%. From Fig. 11 one can see that it takes more than twice the number of NTUs (NTU = 4.0) for a conventional heat exchanger to equal this efficiency at the same capacity rate ratio. Therefore, the linear case illustrates that capacitive heat exchangers are more efficient than conventional ones.

## B. Nonlinear Case

If the physical properties of the gas are strongly dependent on temperature, as they are in this case for the temperatures considered, then it is more accurate to input these relationships into the respective temperature equation. In this case, the appearance of a function of  $T_c$  in Eqs. (38) and (39) creates a nonlinear boundary value problem that can be solved by using the shooting method. In particular, the following formulations can be used for the gas

$$\left. \begin{aligned} \rho_c &= A_1 (1 + B_1 T_c) \\ \mu_c &= A_2 (1 + B_2 T_c) \\ K_c &= A_3 (1 + B_3 T_c) \\ C_{P_C} &= A_4 (1 + B_4 T_c) \end{aligned} \right\} \quad (46)$$

Some of these relationships are also incorporated in the correlation for  $h$ , as given by Eq. (3). A great deal of simplification can be obtained by keeping only the linear terms of the binomial expansion; i.e.,

$$(1 + B_i T_c)^m \cong 1 + m B_i T_c$$

After some algebra, the following set of equations is obtained:

$$\frac{dT_c}{dy} = -\xi_1 \left[ \frac{1 + \xi_2 T_c + \xi_3 T_c^2}{1 + \xi_4 T_c + \xi_5 T_c^2} \right] \left( \frac{3}{r_D} \right) \left( \frac{\kappa}{1 - \kappa} \right) (T_c - T_D) \quad (47)$$

$$\frac{dT_D}{dy} = \xi_6 \left[ \frac{1 + \xi_7 T_c + \xi_8 T_c^2}{1 + \xi_9 T_c} \right] \left( \frac{3}{r_D} \right) (T_D - T_c) \quad (48)$$

with the same boundary conditions as before (Eq. 41). The results appear in Fig. 12. For this nonlinear case the efficiency is 98% as compared to 92% for the linear case; this gives an indication of the increased accuracy of the nonlinear formulation.

#### IV. Conclusion

This report has illustrated by simple examples the nature of the performance of a capacitive heat exchanger. In particular, it was shown that capacitive heat exchangers are more efficient than conventional ones from a thermodynamic point of view. It also pointed out the important role played by the particle volumetric concentration parameter  $\kappa$  in describing the performance of a capacitive heat exchanger. Although the complete problem (Eqs. 5-13) was not attacked, a restricted case (Eqs. 47-48) has shown that the shooting method is capable of obtaining accurate results for such complex, nonlinear boundary value problems.

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## Definition of Symbols

$A$	cross-sectional area of the conduit	$r$	radius
$c_p$	specific heat at constant pressure	$T$	temperature
$c_v$	specific heat at constant volume	$t$	time
$C_d, \overline{C_d}$	drag coefficient	$V$	volume
$\mathcal{D}$	kinetic energy dissipation rate	$v$	velocity
$E$	internal energy	$y$	axial distance for the heat exchanger
$g$	gravitational acceleration	$\beta$	heat transfer parameter
$H$	maximum length (height) of the heat exchanger	$\eta$	loading ratio, equal to $\lambda_D v_D / \lambda_c v_c$
$h$	convective heat transfer coefficient	$\kappa$	volume fraction occupied by particles
$k$	conductive heat transfer coefficient	$\lambda$	species density, equal to volume fraction times material density
$\dot{m}$	mass flow rate	$\mu$	coefficient of viscosity
$N_D$	number of particles per unit volume of mixture	$\rho$	material density
$p$	pressure	$\tau_{ij}$	stress tensor
$Q$	energy sources per unit volume	Suffixes	
$q$	heat per unit volume		
$R$	gas constant		
		$c$	pertains to continuous phase (gas)
		$D$	pertains to dispersed phase (particles)

**Table 1. Summary of equations and unknowns**

Equation type	Equation	Unknowns
Continuity (continuous phase)	$\frac{d}{dy} [(1 - N_D V_D) \rho_c v_c] = 0$ (T1.1)	$N_D, \rho_c, v_c$
Continuity (dispersed phase)	$\frac{d}{dy} (\rho_D N_D v_D) = 0$ (T1.2)	$\rho_D, v_D$
Momentum (continuous phase)	$v_c \frac{dv_c}{dy} = -g - \frac{1}{\rho_c (1 - N_D V_D)} \frac{d}{dy} + \frac{1}{2} \bar{C}_d v_{net}^2 N_D \pi r_D^2$ (T1.3)	$\rho, \bar{C}_d$
Momentum (dispersed phase)	$v_D \frac{dv_D}{dy} = \frac{3}{8} \bar{C}_d (1 - N_D V_D) \frac{\rho_c}{\rho_D} \frac{v_{net}^2}{r_D} - g \left( 1 - \frac{\rho_c}{\rho_D} \right)$ (T1.4)	—
Energy (continuous phase)	$\rho_c v_c c_{p_c} \frac{dT_c}{dy} = +h \left( \frac{4 \pi N_D r_D^2}{1 - N_D V_D} \right) (T_D - T_c)$ (T1.5)	$T_D, T_c, h, c_{p_c}$
Energy (dispersed phase)	$\rho_D v_D c_{p_D} \frac{dT_D}{dy} = \frac{-3h}{r_D} (T_c - T_D)$ (T1.6)	$c_{p_D}$
State	$\rho = \rho_e R T_c$ (T1.7)	—
Drag coefficient correlation	$C_d = f_1 (N_{Re})$ (T1.8)	$\mu_c$
Nusselt nr. correlation	$h = f_2 (N_{Re})$ (T1.9)	—
Viscosity correlation	$\mu = f_3 (T_c)$ (T1.10)	—
$c_{p_c}$ correlation	$c_{p_c} = f_4 (T_c)$ (T1.11)	—
$c_{p_D}$ correlation	$c_{p_D} = f_5 (T_D)$ (T1.12)	—
Density correlation	$\rho_D = f_6 (T_D)$ (T1.13)	—

**Table 2. Dimensionless variables**

$y^* = \frac{y}{H}$	$c_{p_c}^* = \frac{c_{p_c}}{c_{p_D}}$
$r_D^* = \frac{r_D}{H}$	$c_{p_D}^* = 1$
$\nu_D^* = \frac{\nu_D}{\nu_D(H)}$	$k_c^* = \frac{k_c}{k_c(H)}$
$\nu_c^* = \frac{\nu_c}{\nu_D(H)}$	$\rho_c^* = \frac{\rho_c}{\rho_c(0)}$
$T_c^* = \frac{T_c}{T_D(H)}$	$\rho_D^* = 1$
$T_D^* = \frac{T_D}{T_D(H)}$	$N_D^* = \frac{N_D}{N_D(H)}$
$p^* = \frac{p}{p(0)}$	

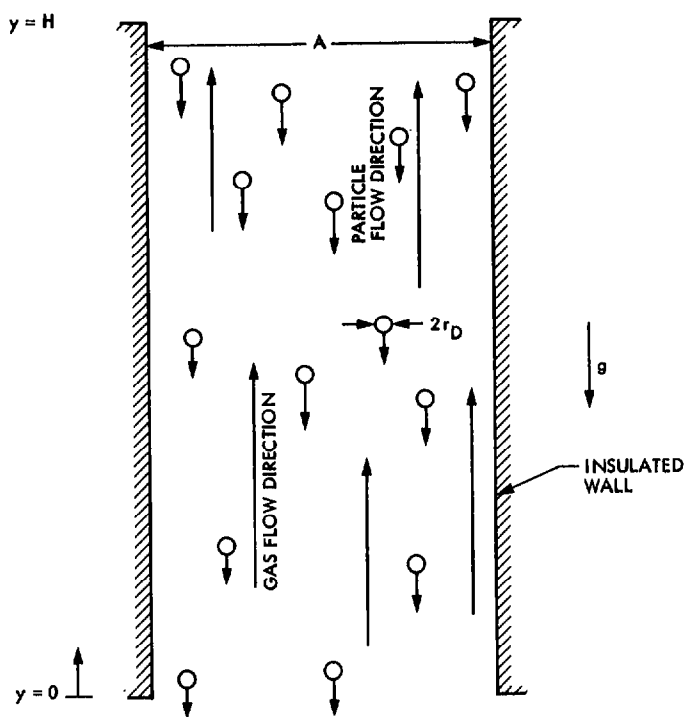


Fig. 1. Geometry of a capacitive heat exchanger

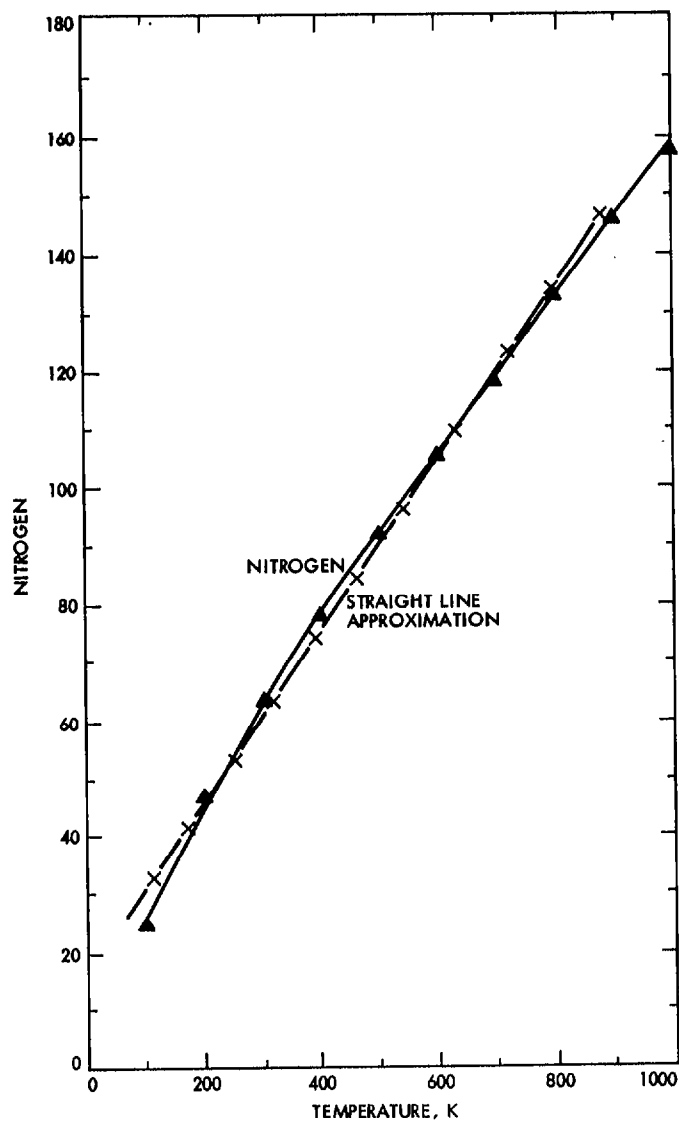


Fig. 2. Thermal conductivity coefficient vs temperature for various gases



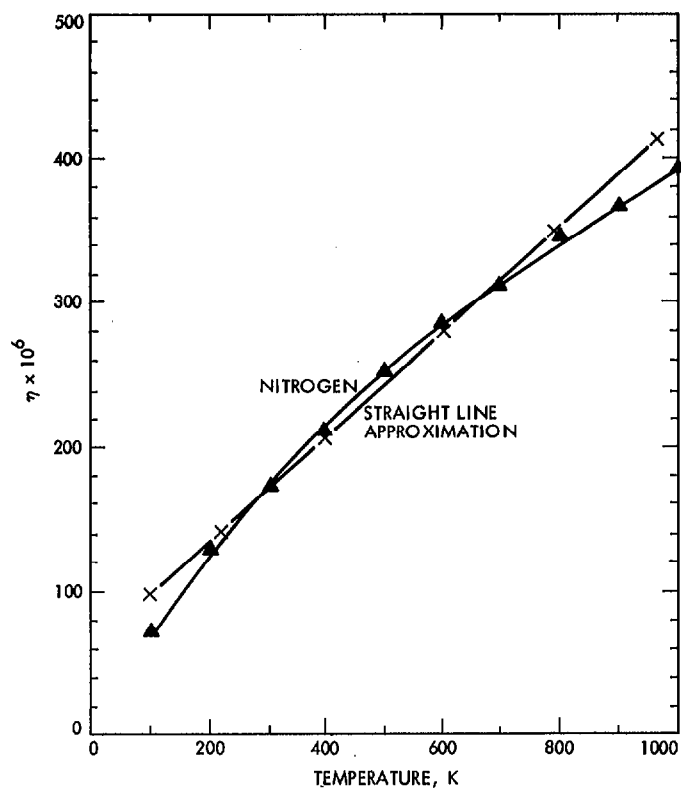


Fig. 3. Viscosity coefficient vs temperature for various gases

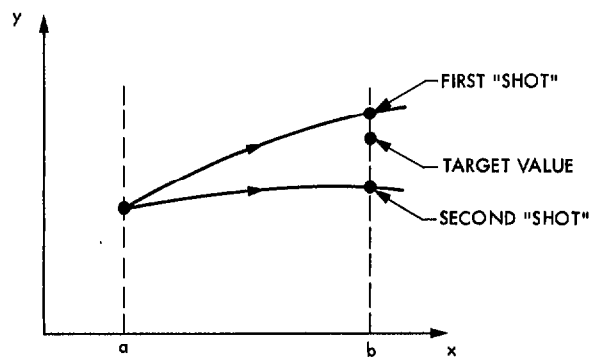


Fig. 4. General principles of the simple shooting method

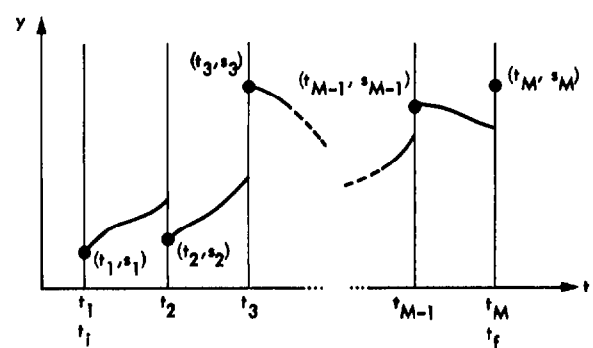


Fig. 5. Multiple shooting method

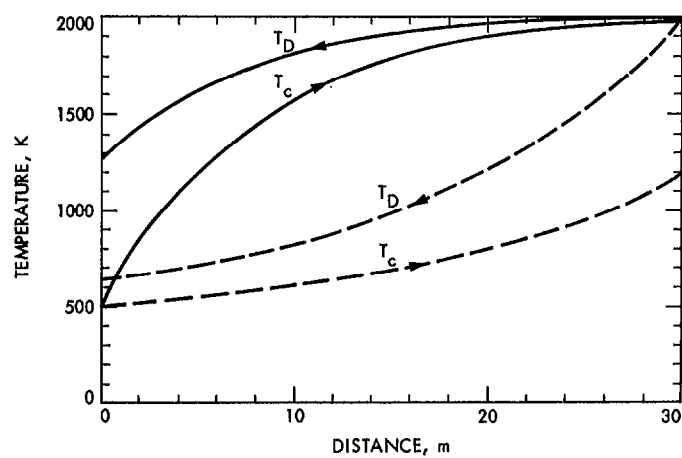


Fig. 6. Variations of temperatures with distance

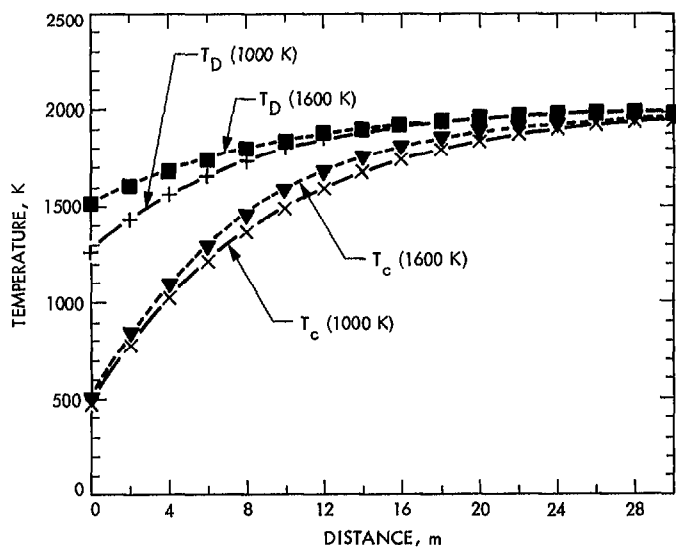


Fig. 7. Effect of physical properties and axial temperature variations

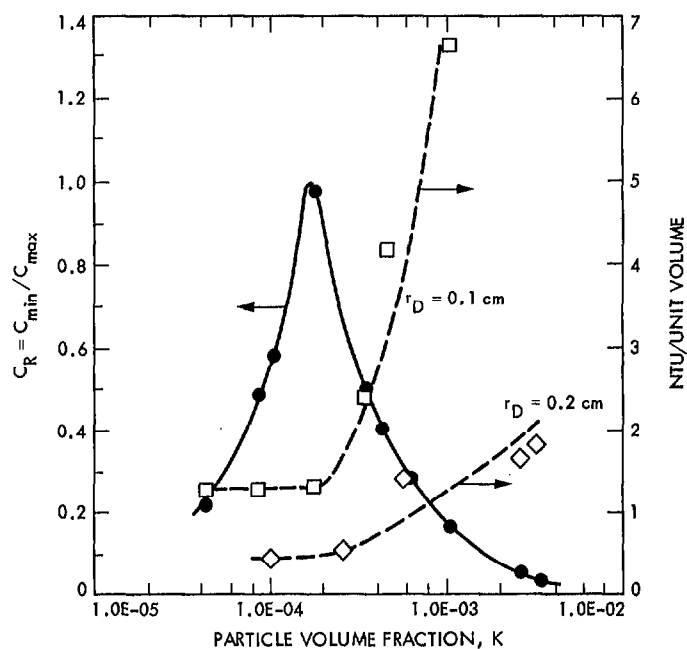


Fig. 9.  $C_R$ , NTU vs  $\kappa$

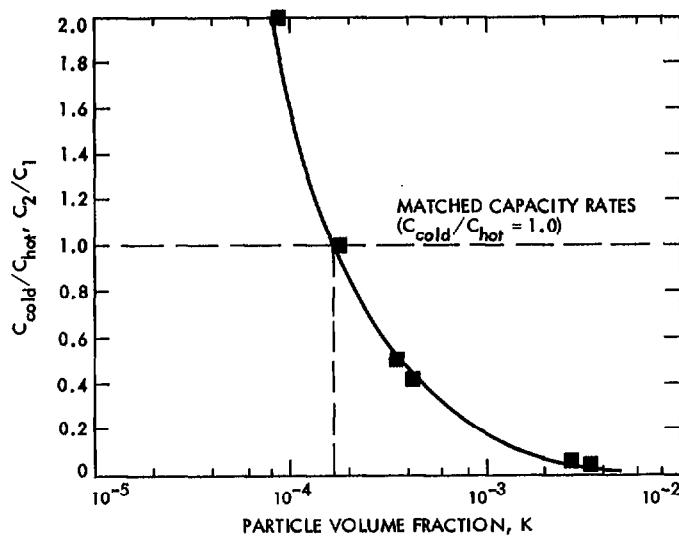


Fig. 8.  $C_2/C_1$  vs  $\kappa$

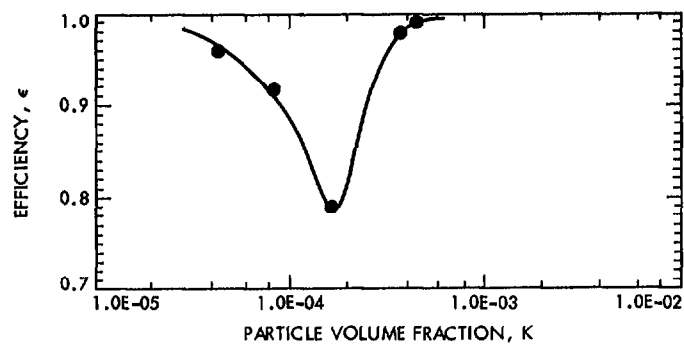


Fig. 10.  $\epsilon$  vs  $\kappa$

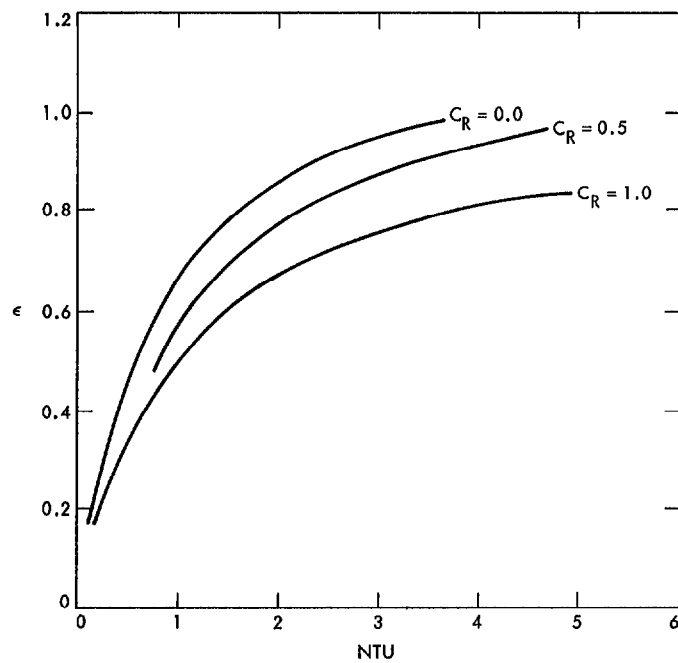


Fig. 11.  $\epsilon$ -NTU relationships for a counterflow heat exchanger

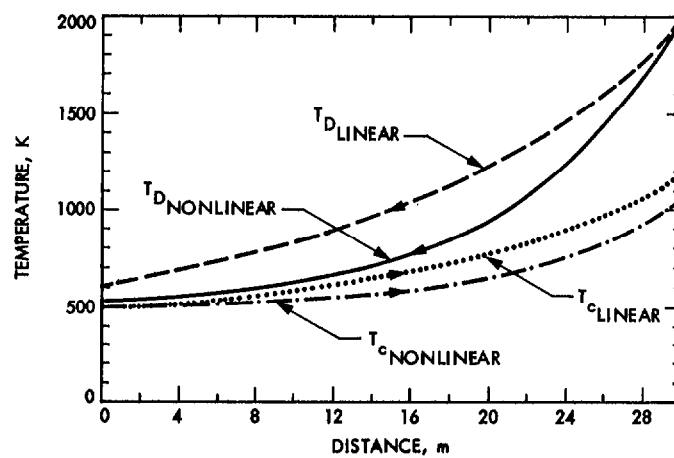


Fig. 12. Axial variations of temperatures in the nonlinear case